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Fiorenzo Franceschini & Domenico Maisano

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Design decisions: concordance of designers and effects of the Arrow's theorem on the collective preference ranking

Fiorenzo Franceschini¹ · Domenico Maisano¹Received: 2 January 2019 / Accepted: 8 March 2019 / Published online: 16 March 2019
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Abstract

The problem of collective decision by design teams has received considerable attention in the scientific literature of engineering design. A much debated problem is that in which multiple designers formulate their individual preference rankings of different design alternatives and these rankings should be aggregated into a collective one. This paper focuses the attention on three basic research questions: (1) “How can the degree of concordance of designer rankings be measured?”, (2) “For a given set of designer rankings, which aggregation model provides the most coherent solution?”, and (3) “To what extent is the collective ranking influenced by the aggregation model in use?”. The aim of this paper is to present a novel approach that addresses the above questions in a relatively simple and agile way. A detailed description of the methodology is supported by a practical application to a real-life case study.

Keywords Engineering design · Collective decision-making · Design teams · Preference ranking · Collective ranking · Degree of concordance

1 Introduction

A general problem, which may concern many design contexts, is to aggregate multiple rankings of different design alternatives into a collective (social) one. This problem may concern the early design stages, in which designers often have conflicting opinions (Fu et al. 2010; Frey et al. 2009; Hoyle and Chen 2011; Keeney 2009; Weingart et al. 2005).

Let us consider the case in which m designers (or, more in general, *decision-making agents*: D_1 – D_m) formulate their corresponding m preference rankings among n design alternatives (or, more in general, *objects*: O_1 , O_2 , O_3 , etc.). It is assumed that the preference rankings of designers are *complete*, i.e., all designers are able to rank all the alternatives of interest, without omitting any of them.

Each ranking can be decomposed into paired-comparison statements such as $O_1 > O_2$, $O_1 \sim O_2$, and $O_2 > O_1$, where

symbols “ $>$ ” and “ \sim ”, respectively, mean “strictly preferred to” and “indifferent to”. Additionally, designer rankings—which will hereafter also be referred to as “preference profiles”—may be more or less *concordant* with each other.

The collective ranking is supposed to reflect the m -rankings as much as possible, even in the presence of diverging preferences. For this reason, it can also be defined as *social*, *consensus* or *compromise* ranking (Cook 2006; Herrera-Viedma et al. 2014; Franceschini et al. 2016).

The aggregation problem of interest may also take into account the importance of designers, which could not necessarily be equal for all of them. However, for the purpose of simplicity, in the rest of the paper designers will be assumed to be equally important.

Decision-making problems based on rankings are very diffused in the scientific literature for two reasons: (1) rankings are probably the most intuitive and effective way to represent preference judgments of design alternatives, and (2) rankings do not require a common reference scale—neither numeric, linguistic or ordinal—to be shared by designers (Yager 2001; Chen et al. 2012).

Many approaches have been proposed in the literature to address the problem of aggregating rankings by multiple decision-making agents (Arrow 2012; Rayanaud 1986; Franssen 2005; Cook 2006; Hazelrigg 1999; Frey et al.

✉ Fiorenzo Franceschini
fiorenzo.franceschini@polito.it
Domenico Maisano
domenico.maisano@polito.it

¹ DIGEP (Department of Management and Production Engineering), Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy

2010; Katsikopoulos 2012; Ladha et al. 2003; Reich 2010). In general, different aggregation models may lead to different collective rankings of the alternatives under consideration (social choice) (Arrow and Raynaud 1986; McComb et al. 2017). The fact remains that it is not easy to identify the model that provides the collective ranking that best reflects the m -rankings. This paper will try to address this problem.

In recent years, a long and passionate debate on the applicability and effects of the Arrow's theorem on design decisions has characterized the engineering design literature (Arrow 2012; Raynaud 1986; Reich 2010; Hazelrigg 1996, 1999, 2010; Scott and Antonsson 1999; Franssen 2005; Yeo et al. 2004; McComb et al. 2017). In a nutshell, this theorem establishes the impossibility of a generic aggregation model to provide a solution (i.e., a collective ranking) that is always fair.¹

In line with the hypotheses of McComb et al. (2017), this paper is concerned with team decisions in the early design stages. Since design outlines are not yet well defined at this stage, the debate on the most relevant design criteria and methods to quantify/prioritize them is potentially very intense (Weingart 2005; Kaldate 2006; McComb et al. 2015, 2017). Although there is a substantial agreement on the criteria that may condition the design process, the selection of design alternatives is generally driven by the different personal experience of designers (Dwarakanath and Wallace 1995).

It is then necessary to adopt appropriate aggregation models to “transform” the decisions of individual designers into a collective design decision. In this scenario, the Theorem of Arrow finds grounds for its application, with some important practical effects (Hazelrigg 1999; Saari and Sieberg 2004; Franssen 2005; Jacobs et al. 2014). The first effect—as the theorem itself shows—is that there is no aggregation model that can always satisfy several properties, also known as *fairness* criteria (Arrow 2012; Fishburn 1973a, Nisan et al. 2007; Saari 2011a, b):

1. *Unrestricted domain (universality)* The aggregation model is defined for problems characterized by any number of decision-making agents, any number of alternatives and any composition of preference rankings over alternatives.
2. *Non-dictatorship* The aggregation model does not simply return the very same preference ranking of a specific decision-making agent (i.e., the so-called *dictator*).
3. *Independence of irrelevant alternatives (IIA)* The collective preference between alternatives x and y must depend

solely on the relative preferences between x and y in the individual rankings.

4. *(Weak) monotonicity* If any decision-making agent modifies his or her preference ranking by promoting a certain alternative, then the collective preference ranking should respond only by promoting the same alternative or remaining unchanged.
5. *Pareto efficiency (unanimity)* If all decision-making agents prefer x to y , then the collective ranking must also prefer x to y .

The second effect of the Arrow's theorem is that one cannot establish *ex ante* the aggregation model that most consistently reflects the preference rankings by decision-making agents. Several quantitative attempts of measuring this *coherence* (or consistency) are discussed (Chiclana et al. 2002; Franceschini and Maisano 2015a, 2017). We also recall McComb et al. (2017), who hypothesized a relationship between the concept of aggregation-model *fairness* and that of *implicit agreement*. The most coherent aggregation model can generally be identified *ex post*, since it may depend on the preference profile.

The third effect of Arrow's theorem is probably less intuitive than the previous ones. Since each aggregation model is inherently “imperfect” (consistently with the afore-illustrated first effect), the resulting collective ranking will also be imperfect. The imperfection of the collective ranking may partly depend on (1) the preference profile and (2) the aggregation model. Unfortunately, it is not easy to separate the impact of these two factors. This concept has been elegantly summarized in a statement by Saari (Decision and Elections 2001, p. 13), in the field of voting theory: “...the winner of an election may more accurately reflect the choice of a decision procedure rather than the view and preferences of voters”.

The scientific literature includes a wide number of aggregation models, which have been analyzed extensively from the perspective of different axioms and properties (Arrow 2012; Fishburn, 1973; Cook 2006; Saari 2011a, b; Nurmi 2012). Some researches demonstrate the effectiveness of specific aggregation models, even though they do not satisfy some of the Arrow's fairness criteria. For instance, Dym et al. (2002) showed that, although the Borda aggregation model may not satisfy the IIA condition, this rarely affects the most preferred alternatives. In other words, Arrow's theorem poses a considerable theoretical problem, but the practical implications are often not so worrisome.

Another research by See and Lewis (2006) proposes a structured approach to avoid severe theoretical conflicts. The paper by Katsikopoulos (2009) expressed the need for greater clarity in the discussion of design decision models; a distinction between the concepts of coherence and correspondence is necessary to structure design activities.

¹ This concept will be clarified later.

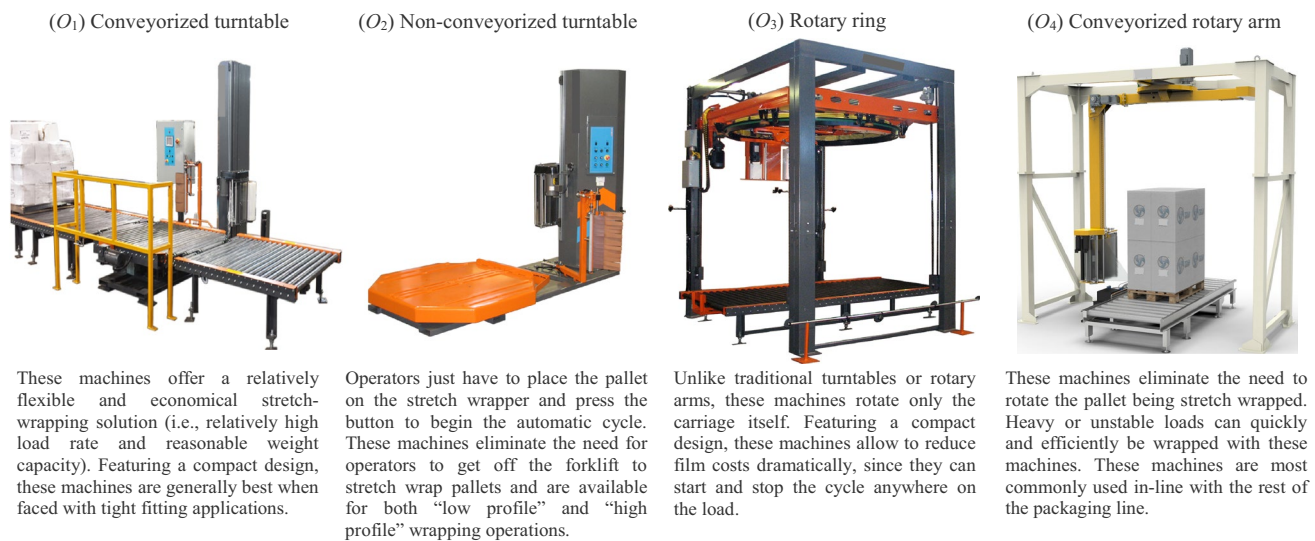


Fig. 1 Schematic representation and short description of four design concepts of automatic stretch-wrapping machines

Coherence is used to characterize the internal consistency of a model, while *correspondence* refers to the representation of the external performance. Additionally, Katsikopoulos (2009) suggested that the arguments by Scott and Antonsen (1999) are rooted in the framework of correspondence, while the arguments introduced by Franssen (2005) in that of coherence. Jacobs et al. (2014) recognized several additional issues on the concepts of uncertainty, comparability and measurability concerned with aggregation models.

Considering the above implications of the Arrow’s theorem, the choice of the aggregation model depends on the specific objective(s) of the decision-making team (Dong et al. 2004; Cagan and Vogel 2012). Selecting an adequate aggregation model is not an easy task, since this choice can importantly affect the collective ranking (Li et al. 2007; Paulus et al. 2011; Franceschini et al. 2019). For the purpose of example, possible selection strategies could be:

1. designers select only the most preferred alternative;
2. designers select the two most preferred alternatives;
3. designers reject the least preferred alternative(s);
4. ...

In this paper, we try to face three basic research questions:

- “How can the degree of concordance of designer rankings be measured?”;
- “For a given set of designer rankings, which aggregation model provides the most coherent solution?”;²

² I.e., the solution that best reflects designer rankings.

- “To what extent, is the collective ranking influenced by the aggregation model in use?”

The remainder of this article is organized into six sections. Section 2 introduces a case study concerning the design of an automatic pallet stretch-wrapping machine, which will accompany the theoretical description of the proposed methodology. Section 3 focuses on the estimation of the concordance of designer rankings, presenting some popular aggregation models. Sections 4 propose a novel method for testing the coherence of the collective ranking with designer rankings, trying to quantify the influence of the aggregation model in use. Section 5 summarizes the original contributions of this paper, practical implications, limitations and suggestions for future research.

2 Real-life case study

The conceptual description of the proposed approach will be accompanied by a practical application to a real-life case study, which is illustrated below. Suppose a company designs and manufactures automatic machines for stretch-wrapping pallets. Four design concepts (O_1 – O_4 , i.e., the alternatives or objects of the problem) of automatic machines have been generated by a team of designers during the conceptual design phase (see the short description in Fig. 1):

- (O_1) conveyORIZED turntable;
- (O_2) non-conveyORIZED turntable;
- (O_3) rotary ring;
- (O_4) conveyORIZED rotary arm.

Table 1 Preference profile provided by ten design engineers (D_1 – D_{10}) for the four design concepts (O_1 – O_4)

Designer	Preference ranking
D_1	$O_1 > O_2 > O_3 > O_4$
D_2	$O_1 > O_2 > O_3 > O_4$
D_3	$O_1 > O_3 > O_4 > O_2$
D_4	$O_1 > O_4 > O_3 > O_2$
D_5	$O_1 > O_4 > O_3 > O_2$
D_6	$O_3 > O_2 > O_4 > O_1$
D_7	$O_3 > O_2 > O_4 > O_1$
D_8	$O_4 > O_2 > O_3 > O_1$
D_9	$O_4 > O_2 > O_3 > O_1$
D_{10}	$O_4 > O_2 > O_3 > O_1$

The objective is to evaluate the afore-mentioned design concepts in terms of *user friendliness*, i.e., a measure of the ease of use of a machine, which generally implies a certain level of automation and a good user interface (Önüt et al. 2008). Some of the factors that can positively influence user friendliness are: (1) the ability of a machine to adapt to loads with different mass, (2) the ability to be integrated within various production lines, or (3) the reduced set-up operations.

A collective judgment should be obtained by merging the individual (subjective) evaluations of ten design engineers (i.e., the decision-making agents of the problem: D_1 – D_{10}). The case study actually involved a team of designers in an important company based in north-western Italy. For reasons of confidentiality, the company is kept anonymous.

The preference profile provided for the four design concepts is shown in Table 1.

3 Concordance of designers

3.1 The W coefficient

The problem of evaluating the degree of concordance/agreement of a set of preference rankings (preference profile variability) can be traced back to the so-called problem of m -rankings. In this context, Kendall and Smith (1939) proposed the variance of the sum of the rank positions as a measure of this concordance. Precisely, a coefficient of concordance W can be defined as (Kendall and Smith 1939; Kendall 1962; Fishburn 1973b; Legendre 2005, 2010):

$$W = \frac{S}{\frac{m^2(n^3-n)}{12}}. \quad (1)$$

The term S in Eq. (1) corresponds to the observed sum of squares of the deviations of the sums of the rank positions with respect to the mean value:

$$S = \sum_{i=1}^n (R_i - \bar{R})^2. \quad (2)$$

The term $m^2(n^3-n)/12$ is the maximum possible value of S , which occurs in the case of complete unanimity in the rankings ($W = 1$).

The first term R_i in the brackets of Eq. (2) is the sum of the rank positions for the i -th object:

$$R_i = \sum_{j=1}^m r_{ij}, \quad (3)$$

in which terms r_{ij} represent the rank of object O_i , according to the j -th designer, n the total number of objects and m represents the total number of rankings. The second term in the brackets in Eq. 2 is the mean value of the R_i values:

$$\bar{R} = \frac{1}{n} \sum_{i=1}^n R_i, \quad (4)$$

where W is defined in the dominion $[0, 1]$, $W = 0$ the absence of concordance, while $W = 1$ indicates the complete concordance (unanimity). By replacing Eqs. (2), (3) and (4) within Eq. (1), the following expression can be obtained:

$$W = \frac{12 \sum_{i=1}^n R_i^2 - 3m^2n(n+1)^2}{m^2n(n^2-1)}. \quad (5)$$

The coefficient W may be slightly modified if, apart from strict-preference relationships, preference rankings also include tied values, characterized by *indifference* relationships (e.g., $O_1 \sim O_2$) (Kendall 1962; Legendre 2010).

The effect of ties is to reduce the value of W ; however, this effect is small unless the number of ties is large. To correct for ties, we introduce the following correction factor:

$$T_j = \sum_{i=1}^{g_j} (t_i^3 - t_i); \quad \forall j = 1, \dots, m, \quad (6)$$

where t_i is the number of tied ranks in the i -th group of tied ranks (where a group is a set of values having the same (tied) rank), and g_j is the number of groups of ties in the set of ranks (ranging from 1 to n) for the j -th designer. Thus, T_j is the correction factor required for the set of ranks related to the j -th designer. Note that if there are no tied ranks for the j -th designer, then $T_j = 0$.

With the correction for ties, the formula of W (Eq. 5) becomes that of W_T :

$$W_T = \frac{12 \sum_{i=1}^n R_i^2 - 3m^2n(n+1)^2}{m^2n(n^2-1) - m \sum_{j=1}^m T_j}, \quad (7)$$

Table 2 Ranks for the four design concepts (O_1 , O_2 , O_3 and O_4), obtained from the designer rankings in Table 1

Designer	Rank positions			
	O_1	O_2	O_3	O_4
D_1	1	2	3	4
D_2	1	2	3	4
D_3	1	4	2	3
D_4	1	4	3	2
D_5	1	4	3	2
D_6	4	2	1	3
D_7	4	2	1	3
D_8	4	2	3	1
D_9	4	2	3	1
D_{10}	4	2	3	1
R_{O_i}	25	26	25	24

where $\sum_{j=1}^m T_j$ is the sum of the values of T_j . W_T is also defined in the dominion [0, 1].

Returning to the case study concerning the four design concepts of automatic stretch-wrapping machines, Table 2 shows the calculation of their ranks.

Since there are no tie-break conditions, we may immediately calculate the coefficient of concordance for the m -rankings using Eq. (5):

$$W^{(m)} = \frac{12(25^2 + 26^2 + 25^2 + 24^2) - 3 \times 10^2 \times 4(4 + 1)^2}{10^2 4(4^2 - 1)} = 0.004 = 0.4\%. \quad (8)$$

The subscript “ (m) ” indicates that this indicator is determined by considering the m -rankings of the designers. The low value of $W^{(m)}$ denotes a very low concordance of the m -designer rankings in the evaluation of the four design concepts. Section 1 (in Appendix) analyzes the W indicator from a statistical point of view and provides a practical method for testing its significance.

3.2 Choice of the aggregation model

The choice of the aggregation model usually reflects the strategy of a team of designers. The aggregation models are inspired by the need to take into account all information provided by designers, ensuring fairness criteria in the treatment of information (McComb et al. 2017). With reference to the case study (see Sect. 2), we now consider four different aggregation models, inspired by four corresponding strategies of the design team.

1. Best of the best model (BoB or standard plurality vote)

For each designer, the most preferred design concept is selected. With reference to Table 1, O_1 gets five preferences,

O_2 gets zero preferences, O_3 gets two preferences and O_4 gets three preferences. The collective ranking is, therefore, $O_1 > O_4 > O_3 > O_2$ and the preferred design concept is O_1 .

2. Best two models (BTW or vote for two)

For each designer, the two most preferred design concepts are selected. With reference to Table 1, O_1 gets five preferences, O_2 gets seven preferences, O_3 gets three preferences and O_4 gets five preferences.

The collective ranking is, therefore, $O_2 > O_1 \sim O_4 > O_3$ and the preferred design concept is O_2 . In this case, we observe an equivalence condition between O_1 and O_4 .

3. Best three models (BTH or vote for three)

For each designer, the three most preferred design concepts are selected (i.e., the worst solution of each designer is not considered). In this case, O_1 gets five preferences, O_2 gets seven preferences, O_3 gets ten preferences and O_4 gets eight preferences. The collective ranking is, therefore, $O_3 > O_4 > O_2 > O_1$ and the preferred design concept is, therefore, O_3 .

4. Borda count model (BC)

For each rank position, a score is assigned (Borda 1781): respectively, 3 points to the first one, 2 points to the second, 1 point to the third, and 0 points to the fourth one. With reference to Table 1, we obtain:

$$\begin{aligned} BC(O_1) &= 5 \times 3 + 0 \times 2 + 0 \times 1 + 5 \times 0 = 15; & BC(O_2) &= 14, \\ BC(O_3) &= 15; & BC(O_4) &= 16. \end{aligned} \quad (9)$$

$BC(O_1)$, $BC(O_2)$, $BC(O_3)$ and $BC(O_4)$ being, respectively, the Borda scores calculated for the four design concepts. The collective ranking is, therefore, $O_4 > O_1 \sim O_3 > O_2$ and the preferred design concept is O_4 . Also in this case, we observe an equivalence condition between O_1 and O_3 .

The four aggregation models reflect different selection strategies, leading to four different collective rankings and four corresponding preferred design concepts. Which collective ranking best reflects the m -designer rankings?

From a conceptual point of view, all these models deserve equal dignity, since they are all potentially plausible and justifiable. However, the fact remains that they may importantly affect the solution. The following section will try to answer the above question.

Finally, we point out that the four aggregation models considered above can be classified as “positional models” and allow to determine a collective ranking in a single round. Alternatively, we could have used so-called “multi-round” aggregation models, in which the

Table 3 m -rankings and corresponding rank positions of (1) the four design concepts (O_1 – O_4) formulated by ten designers (D_1 – D_{10}) (see Table 1), and (2) the collective ranking obtained through the BoB model

Designer	Preference ranking	Rank positions			
		O_1	O_2	O_3	O_4
D_1	$O_1 > O_2 > O_3 > O_4$	1	2	3	4
D_2	$O_1 > O_2 > O_3 > O_4$	1	2	3	4
D_3	$O_1 > O_3 > O_4 > O_2$	1	4	2	3
D_4	$O_1 > O_4 > O_3 > O_2$	1	4	3	2
D_5	$O_1 > O_4 > O_3 > O_2$	1	4	3	2
D_6	$O_3 > O_2 > O_4 > O_1$	4	2	1	3
D_7	$O_3 > O_2 > O_4 > O_1$	4	2	1	3
D_8	$O_4 > O_2 > O_3 > O_1$	4	2	3	1
D_9	$O_4 > O_2 > O_3 > O_1$	4	2	3	1
D_{10}	$O_4 > O_2 > O_3 > O_1$	4	2	3	1
BoB model	$O_1 > O_4 > O_3 > O_2$	1	4	3	2
R_{O_i}		26	30	28	26

alternatives can be phased out in different rounds (Saari 2011a, b; Bormann and Golder 2013).

4 Coherence of the collective ranking

4.1 The $W^{(m+1)}$ technique

This approach follows the conceptual scheme seen in Sect. 3.1. The idea is to analyze the level of coherence of the m -designer rankings and the collective ranking resulting from the application of the aggregation model, constructing the new coefficient of concordance $W^{(m+1)}$. As an example, Table 3 shows the construction of $W^{(m+1)}$ for the BoB aggregation model.

The overall coefficient of concordance $W_{\text{BoB}}^{(m+1)}$ for the BoB model can be obtained by applying Eq. (8) to the 11 rankings in Table 3:

$$W_{\text{BoB}}^{(m+1)} = \frac{12(26^2 + 30^2 + 28^2 + 26^2) - 3 \times (10 + 1)^2 \times 4(4 + 1)^2}{(10 + 1)^2 4(4^2 - 1)} = 0.0182 = 1.82\% . \quad (10)$$

We observe that $W_{\text{BoB}}^{(m+1)} > W^{(m)}$, i.e., the overall degree of concordance between the (10 + 1) rankings has increased. This output appears reasonable since the collective ranking resulting from the BoB model is a “composition” of the ten previous rankings and is, therefore, supposed to reflect them all.

Repeating the construction for the other aggregation models (BTW, BTH and BC), results in Table 4 are obtained. The same table compares the $W_i^{(m+1)}$ values with the respective

Table 4 $W_i^{(m+1)}$ values for the four aggregation models (i : BoB, BTW, BTH, BC) compared to the relevant $W^{(m)}$ values

Aggregation model	$W_i^{(m+1)}$ (%)	$W^{(m)}$ (%)
Best of the best (BoB)	1.82	0.4
Best two (BTW)	0.58	0.4
Best three (BTH)	1.48	0.4
Borda count (BC)	2.00	0.4

Table 5 New m -rankings and corresponding rank positions of the four design concepts (O_1 – O_4) formulated by ten designers (D_1 – D_{10})

Designer	Preference ranking	Rank positions			
		O_1	O_2	O_3	O_4
D_1	$O_1 > O_2 > O_3 > O_4$	1	2	3	4
D_2	$O_1 > O_2 > O_3 > O_4$	1	2	3	4
D_3	$O_1 > O_2 > O_3 > O_4$	1	4	2	3
D_4	$O_1 > O_2 > O_3 > O_4$	1	4	3	2
D_5	$O_1 > O_2 > O_3 > O_4$	1	4	3	2
D_6	$O_1 > O_3 > O_4 > O_2$	4	2	1	3
D_7	$O_1 > O_4 > O_3 > O_2$	4	2	1	3
D_8	$O_1 > O_4 > O_3 > O_2$	4	2	3	1
D_9	$O_3 > O_2 > O_4 > O_1$	4	2	3	1
D_{10}	$O_3 > O_2 > O_4 > O_1$	4	2	3	1
R_{O_i}		26	30	28	26

$W^{(m)}$ value (the same for all aggregation models) for the four aggregation models of interest.

It can be noticed that the Borda count model obtains the highest value of $W_i^{(m+1)}$; it is, therefore, the aggregation model with the collective ranking that best fits with the m -rankings.

4.2 Further considerations on the $W^{(m+1)}$ technique

If designer rankings are very *polarized* (i.e., they are characterized by a relatively low variability), we expect that the

output of different aggregation models converges towards a common collective ranking, e.g., in the extreme case in which all the m -designer rankings coincide, the four aggregation models would converge into that very specific (collective) ranking.

As a pedagogical example, let us consider a new problem characterized by the set of m -rankings in Table 6. With reference to Table 1, we imagine that the three designers

who indicated the ranking $O_4 > O_2 > O_3 > O_1$ (i.e., D_8 , D_9 and D_{10}) converted their preference into the ranking $O_1 > O_2 > O_3 > O_4$; see the corresponding rank positions on the right-hand side of Table 5.

In this case, the $W^{(m)}$ is supposed to increase with respect to that of Table 1, due to the reinforced polarization of preferences. Applying Eq. (5), we obtain:

$$W^{(m)} = \frac{12(16^2 + 26^2 + 25^2 + 33^2) - 3 \times 10^2 \times 4(4 + 1)^2}{10^2 4(4^2 - 1)} = 0.29 = 29\%. \quad (11)$$

Not surprisingly, this value is conspicuously higher than that in Eq. (8).

Table 6 summarizes the collective rankings resulting from the m -rankings in Table 5, for each (i -th) aggregation model and corresponding $W_i^{(m+1)}$ and $W^{(m)}$ values.

Even in this second scenario, the Borda Count model shows the highest coefficient of concordance. This is not a general rule, since—when changing the preference profile—the model that best reflects it may change.

In addition, we note that the condition $W_i^{(m+1)} \geq W^{(m)}$ holds for each of the four aggregation models, when considering both the preference profiles in Tables 2 and 5. However, this is not a general rule, as shown by the counterexample in Table 7.

Suppose we use a hypothetical aggregation model (H -model) that provides the following collective ranking $O_4 > O_3 > O_2 > O_1$, which is the opposite to those by the two designers of interest. In this case, it can be verified that $W^{(m)} = 1$, while

$$W_H^{(m+1)} = \frac{12(6^2 + 7^2 + 8^2 + 9^2) - 3 \times (2 + 1)^2 \times 4(4 + 1)^2}{(2 + 1)^2 4(4^2 - 1)} = 0.111 = 11.1\%. \quad (12)$$

Although this exercise may seem somewhat implausible, it shows that—from a formal point of view—the condition $W_H^{(m+1)} \geq W^{(m)}$ does not necessarily hold.

The $W^{(m+1)}$ technique considers the coefficient of concordance $W^{(m)}$ as the fulcrum for checking the concordance of the m -rankings and the coherence with the collective

Table 6 Collective rankings provided by four different aggregation models (BoB, BTW, BC) for the (m) designer rankings reported in Table 5, and corresponding $W_i^{(m+1)}$ and $W^{(m)}$ values

Aggregation model	Collective ranking	$W_i^{(m+1)}$ (%)	$W^{(m)}$ (%)
Best of the best (BoB)	$O_1 > O_3 > O_2 \sim O_4$	32.6	29
Best two (BTW)	$O_1 > O_2 > O_3 > O_4$	33.2	29
Best three (BTH)	$O_3 > O_1 > O_2 > O_4$	30.6	29
Borda count (BC)	$O_1 > O_3 > O_2 > O_4$	33.5	29

Table 7 m -rankings and corresponding rank positions of (1) four design concepts (O_1 – O_4) formulated by two designers (D_1 , D_2), and (2) the collective ranking obtained through a hypothetical aggregation model (H -model)

Designer	Preference ranking	Rank positions			
		O_1	O_2	O_3	O_4
D_1	$O_1 > O_2 > O_3 > O_4$	1	2	3	4
D_2	$O_1 > O_2 > O_3 > O_4$	1	2	3	4
Collective ranking by the H -model	$O_4 > O_3 > O_2 > O_1$	4	3	2	1

ranking. The same technique can also be applied with other potential coefficients of concordance, e.g., the mean value of the Spearman coefficient (ρ) between the $\binom{m}{2}$ possible pairs of designers (Kendall 1962).

4.3 Impact of the aggregation models

This section deals with the problem of measuring the impact of the aggregation model in use on the collective ranking. Consistently with the contents of the previous sections, the following synthetic indicator can be considered:

$$b_i^{(m)} = \frac{W_i^{(m+1)}}{W^{(m)}}. \quad (13)$$

It can easily be proved that $b_i^{(m)} \in]0, \infty[$. For a certain set of m -rankings, if $b_i^{(m)} \geq 1$, it means that the i -th aggregation model provides a somehow coherent collective ranking (positive impact). On the contrary, if $b_i^{(m)} < 1$, it means that the i -th aggregation model provides a somehow incoherent collective ranking (negative impact).

In practice, for a specific set of m -rankings, $b_i^{(m)}$ allows to identify the most coherent aggregation model, i.e., that with

$$b^{(m)*} = \max_i b_i^{(m)}. \quad (14)$$

By way of example, Table 8 reports the value of $b_i^{(m)}$ for the four discussed aggregation models, considering the m -rankings in Table 6.

In this case, the BC is the aggregation model that produces the collective ranking that best reflects the m -rankings.

To test the significance of an observed value of $b_i^{(m)}$, one should consider the relevant distribution, which arises from the composition of the two distributions of $W^{(m)}$ and $W_i^{(m+1)}$ (see also Sect 1 in Appendix). Precisely, one can consider the distribution of $b_i^{(m)}$ in the $(n!)^{(m+1)}$ possible sets of rank positions and use it to reject or accept the hypothesis that designer rankings are concordant or not (Kendall 1962).

Table 8 $b_i^{(m)}$ values for the four aggregation models of interest, considering the m -rankings in Table 6

Aggregation model	$W_i^{(m+1)} (\%)$	$W^{(m)} (\%)$	$b_i^{(m)} (\%)$
Best of the best (BoB)	1.82	0.4	4.55
Best two (BTW)	0.58	0.4	1.45
Best three (BTH)	1.48	0.4	3.70
Borda count (BC)	2.00	0.4	5.00

Table 9 $b_i^{(m)}$ values for the four aggregation models of interest, considering the m -rankings in Table 3

Aggregation model	$W_i^{(m+1)} (\%)$	$W^{(m)} (\%)$	$b_i^{(m)}$
Best of the best (BoB)	32.6	29	1.12
Best two (BTW)	33.2	29	1.14
Best three (BTH)	30.6	29	1.06
Borda count (BC)	33.5	29	1.16

As a further example, Table 9 shows the value of $b_i^{(m)}$ for the four aggregation models discussed in Sect. 3.2, related to the (m) designer rankings in Table 3.

Even in this case, the BC is the aggregation model that produces the collective ranking that best reflects the m -rankings.

In practice, $b_i^{(m)}$ can be used as a metric to assess the *relative impact* of two aggregation models to be compared. For a given preference profile, we define the relative impact in the form of a ratio $s_{ij}^{(m)}$ between the $b_i^{(m)}$ values for two generic models (i and j):

$$s_{ij}^{(m)} = \frac{b_i^{(m)}}{b_j^{(m)}} = \frac{W_i^{(m+1)}}{W_j^{(m+1)}}. \quad (15)$$

As an example, for the BoB and BTW models in Table 9 we obtain:

$$s_{\text{BoB, BTW}}^{(m)} = \frac{b_{\text{BoB}}^{(m)}}{b_{\text{BTW}}^{(m)}} = \frac{4.55}{1.45} = \frac{W_{\text{BoB}}^{(m)}}{W_{\text{BTW}}^{(m)}} = 3.14. \quad (16)$$

The relative impact $s_{\text{BoB, BTW}}^{(m)} = 3.14$ shows the greater impact of the BoB model in supporting the choice of O_1 as opposed to that of the BTW model in supporting the choice of O_2 (in fact, the resulting collective rankings are $O_1 > O_4 > O_3 > O_2$ for BoB and $O_2 > O_1 \sim O_4 > O_3$ for BTW, as shown in Sect. 3.2).

Let us now analyze in detail the $b_i^{(m)}$ indicator (Eq. 13). The denominator ($W^{(m)}$) depends exclusively on the m -rankings, while the numerator ($W_i^{(m+1)}$) depends both on the m -rankings and the collective ranking. For problems characterized by a relatively large number (m) of rankings, the

contribution of the collective ranking will, therefore, tend to have a lower “weight”, i.e., $1/(m+1)$.

Moreover, $b_i^{(m)}$ values tend to be higher for problems characterized by relatively discordant m -rankings (and, therefore, lower $W^{(m)}$ values). For example, the $b_i^{(m)}$ values in Table 9 are significantly higher than those in Table 8, as they result from a problem with significantly more discordant m -rankings and, therefore, relatively lower $W^{(m)}$ values.

5 Discussion

This paper proposed a novel method to support the decisions of teams of designers in early design stages, trying to provide a plausible answer to three different research questions: (1) how can the degree of concordance of designer rankings be measured? (2) For a given set of designer rankings, which aggregation model provides the most coherent solution? (3) To what extent is the collective ranking influenced by the aggregation model in use?

The answer to the first research question is represented by the Kendall’s coefficient of concordance W , or by other similar multi-rater coefficients of agreement. The next two research questions were interpreted as consequences of Arrow’s theorem, when deployed in the context of engineering design. In this field, designers often have contrast opinions on alternative design solutions and it is not easy to decide how to aggregate the choices of individual designers into a single collective one. The answer to the second research question was given by a special use of the coefficient W . This coefficient of concordance is applied not only to m -rankings but also to the collective ranking produced by the aggregation method of interest. The new indicator $W^{(m+1)}$ allows a quick comparison between alternative aggregation models, if they are applied to the same specific problem, describing their “effectiveness”.

For the third research question, paraphrasing Arrow’s theorem, it is difficult to understand if and to what extent a certain aggregation model impacts on the collective ranking. A practical answer is given by the $b_i^{(m)}$ indicator, which depicts the ability of a certain aggregation model to provide a plausible synthesis. $b_i^{(m)}$ can also be used to provide a quantitative *metric* for evaluating the *relative impact* of different models.

Having said that, let us now consider the proposed approach in its entirety. This approach is easily implementable and, with few additional adjustments, can also be applied to problems including *partial* preference rankings (Franceschini and Maisano 2018).

The main limitations of the proposed approach are:

- the method considers neither the (possible) uncertainty in designer rankings, nor possible incompleteness in preference rankings;
- the method allows only an *ex post* analysis of the impact of aggregation models.

Regarding the future, we plan to develop an interactive on-line tool for supporting the activity of design teams. In addition, this tool will be used for more structured problems in the field of design for manufacturing and quality engineering/management (e.g., integrating it with the *Quality Function Deployment* and other design support tools) (Olewnik and Lewis 2008; Franceschini et al. 2015b, c; Chen et al. 2017). Finally, we plan to consider problems in which designers may have a different (ordinal or cardinal) weight, for example, due to a different design experience.

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Appendix

Statistical meaning of W

This subsection analyzes the statistical significance of W . In general, the W distributions are available in tabular terms for small values of m and n (Kendall 1962). For higher values, the Fisher distribution can be used:

$$F = \frac{(m-1) \times W^{(m)}}{[1 - W^{(m)}]} \quad (17)$$

with parameters ν_1 and ν_2 defined, respectively, as

$$\begin{cases} \nu_1 = n - 1 - \frac{2}{m} \\ \nu_2 = (m-1) \left(n - 1 - \frac{2}{m} \right) \end{cases} \quad (18)$$

When $n > 7$, $W^{(m)}$ can be described by a chi-square distribution $\chi_r^2 = m(n-1)W^{(m)}$. χ_r^2 is distributed as a χ_{n-1}^2 with $\nu = n - 1$ degrees of freedom.

For example, considering the data in Table 6, where $m = 10$ designers and $n = 4$ design concepts, $W^{(m)} = 0.29$. Applying Eq. (8), it can be obtained:

$$F = \frac{(10-1) \times 0.29}{(1-0.29)} = 3.67. \quad (19)$$

The degrees of freedom are, respectively:

$$\begin{cases} \nu_1 = 4 - 1 - \frac{2}{10} = 2.8 \approx 3 \\ \nu_2 = (10-1) \left(4 - 1 - \frac{2}{10} \right) = 25.2 \approx 25 \end{cases} \quad (20)$$

From the tables of the Fisher distribution for a significance of 5%, it is obtained $F_{5\%;3;25} = 2.99$.

Since $F > F_{5\%;3;25}$, the significance of the coefficient of concordance for the preference profile in Table 6 is confirmed.

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